Core angular momentum and length of day during the 18^{th} and 19^{th} centuries

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Abstract

Currently available estimates of changes in the length of day (LOD) date back to about 1630 and have associated uncertainties of at most ±1 ms after 1700. The salient feature of the LOD data during the 18^{th} and most of the 19^{th} centuries is the absence of robust decadal fluctuations. This is remarkable in view of the large (±4 ms) decade-scale changes that mark the data after about 1850. What could have caused this change in behavior? Because decade changes in the LOD are thought to originate in the Earth's outer core, we consider whether core fluid motions might present a similar behavioral difference at about the same time. Since about 1930 angular momentum carried by the core motions compares well with angular momentum variations of the mantle reflected in the LOD data (provided flow deep inside the core obeys certain simplifying assumptions). This agreement is considered evidence of core-mantle coupling. If indeed the decadal LOD is a faithful proxy for core angular momentum (CAM), the 18^{th} and early 19^{th} century CAM series should also reveal the curious flatness seen in the LOD during that time. We find instead that the pre-1850 CAM series displays oscillatory behavior more pronounced than that evident in the 20^{th} century. The possible implications of this result are considered, yet the high degree of uncertainty in the geomagnetic field (and hence the corresponding CAM) before 1840 renders our results equivocal at best. Currently, efforts are underway by others to better constrain the main field before 1840. We estimate that for the 18^{th} and early 19^{th} centuries the broad scale main field (coefficients up to degree 4) must be determined to within an average error of 3% or less to achieve the 1 ms accuracy that accompanies the LOD series before 1840.

1. Introduction

Motions of the fluid inside Earth's liquid outer core have long been thought to be the source of the geomagnetic field. From the earliest arguments favoring a hydrodynamic origin over a permanently magnetized source, the putative connection between flow and field has evolved to a stage where now numerical core geodynamos are capable of replicating events as rare as polarity reversals [*Glatzmaier and Roberts*, 1995]. Although details of the convection in the core are still unknown, today little doubt remains that dynamo action sustains the field against constant ohmic decay.

Instead, the debate has shifted to other consequences of core convection. Of particular interest to geodesists is the possibility that some fluctuations of mantle angular momentum, as reflected by observed length of day (LOD) changes, originate in the core. Much of the interannual and shorter term variability in the LOD has been successfully tied to exchanges of angular momentum between the atmosphere/oceans and the solid Earth [see, for example, *Hide and Dickey*, 1991]. Yet, LOD changes on the decade scale have proven more difficult to ascribe to a specific source. The time-scale of changes in the oceans or atmosphere is too short to claim responsibility, and that of geologic events is far too long. It has been argued that core flow provides the most likely means of excitation for these irregular motions of the mantle. Core flow is variable on an intermediate time-scale, and changes in the westward drift of the geomagnetic field over the last century are consistent in sign and magnitude with the decade oscillations of the mantle spin rate [*Munk and MacDonald*, 1960].

As likely as it appears that the core plays a role in the decade LOD changes, quantitative assessment of the coupling needed for the core to torque the mantle has proven the weakest link in this argument. Viscous coupling is perhaps safely neglected, but electromagnetic, topographic, and gravitational coupling remain at best poorly constrained alternatives [*Hide* 1969, *Stix and Roberts*, 1984, *Jault and Mouël*, 1989]. If the core and mantle really are coupled, it should be possible to predict changes in the LOD given estimates of axial core angular momentum (CAM). Values of CAM can be represented as excess or deficit LOD resulting from a transfer of core momentum to the overlying mantle (assumed perfectly rigid). Every $6 \times 10^{25}\mathrm{kgm^2s^{-1}}$ of axial momentum transfered produces a 1 ms LOD change. A comparison between observed and predicted LOD (Figure 1) reveals a remarkable similarity, especially after about 1930. While the means for coupling the core to the mantle eludes definition, the conservation of whole Earth angular momentum over at least the last few decades implied by Figure 1 offers the most convincing evidence yet that such coupling across the core-mantle boundary (CMB) can indeed occur.

Figure 1 only shows observed changes in the excess LOD beginning after 1840, but in fact LOD values based on lunar occultations have been recorded since about 1630 [*Stevenson and Morrison*, 1984; *McCarthy and Babcock*, 1986]. These early estimates lack the precision and accuracy of the more recent space-geodetic measurements. Still, their estimated formal uncertainties, especially after 1700 (± 1 ms or less), are not large enough to obscure variability like that seen during the 20th century, if it were to exist (Figure 2).

The salient feature of the LOD data during all of the 18^{th} and half of the 19^{th} centuries is the absence of decadal fluctuations despite a 0.5 year sampling interval used by *McCarthy and Babcock* (*Stevenson and Morrison* use a 5 year interval between 1690 and 1780, and a 1 year interval thereafter). This is remarkable in view of the persistent and often large decade-scale changes that mark the data after that time. *Munk and MacDonald* [1960] first called attention to this unusual feature branding it "an enigmatic phenomenon." Although the 1 ms uncertainty in the 18^{th} century LOD data may be too optimistic, here we accept it at face value while admitting the possibility that some as yet unforeseen systematic errors may have altered the data.

What might explain the pronounced difference in the behavior of the LOD signal that takes place around 1850? Is the LOD dataset indeed *always* a faithful proxy of angular momentum in the core? If not, then what are the implications for the nature of core-mantle coupling? If it is, can the change in the nature of the LOD signal near 1850 tell us anything useful about motions deep inside the core? Before attempting to resolve some of these issues we must first obtain the motions of the core during the 18^{th} and early 19^{th} centuries and examine the LOD changes they imply.

To that end we estimate axial CAM over a 300 year time span that lengthens the CAM series in Figure 1 by an additional 150 years back to 1690. Throughout this report we express CAM in terms of the millisecond changes it can induce in the LOD as a consequence of coupling. No assumptions about the form or efficiency of the coupling are made. Extending the CAM series requires that we first resolve core surface flows between 1690 and 1840, which we do based on a field model for the same period produced by *Bloxham and Jackson* [1992]. These flows then permit estimation of CAM changes before 1840 under the identical set of assumptions invoked to compute core momentum after 1840.

We find that while the observed LOD series flattens out over the period prior to about 1850 evincing no decade fluctuations, the LOD predicted by CAM variations (in units of ms) displays an oscillatory behavior similar to that evident during more recent times. Our result is clouded, however, by the uncertainty in our knowledge of the magnetic field before 1840. Part of the problem rests with the lack of early field intensity data, but the far more insidious difficulty is our poor measure of temporal changes in the field. We present an error analysis indicating that the broad scale 18^{th} and 19^{th} century fields must be known to within an average error of 3% or less to achieve the 1 ms accuracy of the LOD. This is intended to assist efforts currently underway to better constrain the main field at epochs before 1840. Finally, we consider the implications of believing only the geodetic data, only the magnetic data, neither of these, or both of them. Some of these choices have specific consequences for understanding core flow, core-mantle coupling, and historical Earth rotation.

2. Core Flow and Angular Momentum

Resolving fluid motions near the core surface has been the object of intense study since the late 1960's. The primary expectation has been that a solution would yield insight into the dynamo process. In particular, workers have sought to understand the force balance in the core (geostrophic, magnetostrophic or neither) as this dictates which of several competing dynamo states holds within its volume. The desire to explain long term changes in the LOD and secure constraints on lateral temperature variations at the CMB have also spurred core flow studies. Until recently though, these have been of only secondary importance.

The availability of reliable maps of the core radial field dating back several decades has made it possible to invert for fluid motions exhibiting measurable time dependence [*Jackson et al.*, 1993; *Voorhies*, 1995; *Celaya and Wahr*, 1996]. Although flow variability has not shed further light on the core force balance directly, it has allowed significant advances in our understanding of decade LOD changes.

Current attempts to pin decadal accelerations of the mantle on the exchange of angular momentum between it and the core, require that we know CAM and the LOD changes over a time span long enough to assess their correlation. Starting with *Jault et al.* [1988], several authors have taken advantage of core surface flow time-dependence to produce a time series of CAM. Initially *Jault et al.* noticed that the zonal component of this flow varies with latitude exhibiting a pattern that is roughly symmetric about the equator. They surmised that such a pattern might reflect flow deeper within the core arranged in the form of rigidly rotating coaxial cylinders. Such an inference is justified by the work of *Proudman* [1916], *Taylor* [1921] and *Bullard and Gellman* [1954] who considered motion of an inviscid, incompressible fluid inside a rigid rotating container. Collectively these authors showed that under low Rossby, low Ekman number conditions, the only free motions possible are those in which cylindrical shells move rigidly about the rotation axis. In the presence of a magnetic field, *Taylor* [1963] showed that relative rotations of the cylindrical shells will occur until the azimuthal component of the Lorentz torque over the surface of every cylinder vanishes.

Such a flow is invariant along the direction of the spin axis. This simple geometry connects flow at the core surface, where it is determined by inversion of the downward-continued radial magnetic field, to flow everywhere inside the core. Only motion within a cylinder tangent to the inner core is not well resolved by this approach and because its contribution to axial CAM is minimal (it has the shortest moment arm) it is typically neglected (as we have here). Computing the component of angular momentum along the rotation axis, J_z, is then a straightforward procedure. Neglecting both the small radial variation of density, ρ, and the presence of the inner core, J_z ultimately depends only on the azimuthal flow component as follows,

$$J_z = \hat{\mathbf{z}} \cdot \int_V \mathbf{r} \times \mathbf{u} \rho dV = \rho \int_V r \sin\theta u_\phi dV, \qquad (1)$$

where V is the core volume.

The initial report by *Jault et al.* [1988] showed a significant match of recorded LOD and CAM between 1969 and 1987 achieved through this set of arguments. Their results generated great interest and several authors [*Jault and Le Mouël*, 1989; *Jackson et al.* 1993; *Jackson*, 1997a] followed suit eventually extending the CAM series back to about 1840. As these authors note, the simplicity of the core model invoked leaves open the chance that the LOD-CAM correlation may be fortuitous. Yet, if any of the key assumptions was incorrect the correlation would not hold. The degree to which these severe assumptions about the force balance in the core have come to be believed is such that a match between LOD and CAM is now considered a good measure of the quality of the surface flow solution. Some have now even incorporated this coherence as a constraint in the flow inversion itself [*Holme*, 1998]. Recently *Dumberry and Buffet* [1998] examined the approximations employed to obtain CAM from magnetic field models concluding that they are valid for periods of motion longer than a few days. Ironically, progress on this secondary problem in our understanding of the core geodynamo may ultimately help settle the original force balance issue.

3. Core Surface Fluid Motions

Our strategy for resolving core surface flows follows a well established procedure involving an inversion of the radial component of the magnetic induction equation. Under the frozen flux approximation where diffusion of the field through the fluid is ignored [*Roberts and Scott*, 1965] the radial magnetic field B_r evolves according to,

$$\partial_t B_r = -\nabla_s \cdot (\mathbf{u} B_r), \tag{2}$$

where the vector $\mathbf{u}$ is the Eulerian flow velocity and ∇_s is the horizontal gradient operator ($\nabla_s = \nabla - \hat{\mathbf{r}} \cdot \nabla$). Downward continuation of surface field measurements through the mantle (assumed to be an electrical insulator) provides estimates of B_r and $\partial_t B_r$ that are continuous across the CMB and the thin Ekman-Hartman boundary layer that develops at the top of the core. In this way we establish contact between field and flow at the free stream edge in the core, just beneath the mantle, which we need to proceed with the inversion.

By itself, however, (2) is a single scalar equation in three unknowns (two components of horizontal motion and their surface divergence) and as such is insufficient to determine the surface flow uniquely [*Backus*, 1968]. A variety of assumptions about the physical state of the core have been invoked to reduce the ambiguity. Among these are that the flow lacks horizontal divergence [*Roberts and Scott*, 1965; *Whaler*, 1980; *Gubbins*, 1982], is steady [*Gubbins*, 1982; *Voorhies and Backus*, 1985], or geostrophic [*Le Mouël et al.*, 1985; *Benton*, 1985; *Backus and Le Mouël*, 1986]. The relative success of these approximations has been difficult to measure because each gives a different solution [see, e.g., *Bloxham and Jackson*, 1991]. Yet the steady motions hypothesis enjoys several advantages that persuade us to initially favor it alone above the others. It does not rest on additional physical assumptions about the density stratification or force balance within the core, and it removes the ambiguity everywhere on the core surface rather than leaving patches within which the motion remains ambiguous. Perhaps what is most important is that it is also readily testable against magnetic data alone. Of course, over time spans as long as a century we cannot expect the flow to be steady - especially if we believe that the core and mantle exchange momentum on the decade time scale. Over so much time magnetic diffusion is also not safely neglected. Our approach then is to make several staggered inversions, each one spanning only ten years of magnetic data, and then collect these solutions to obtain a single time-varying flow model valid for 300 years.

Point estimates of B_r and $\partial_t B_r$ at the CMB are subject to great uncertainty [*Gubbins and Bloxham*, 1985]. Rather than invert for equally meaningless local flow velocities, we recast (2) using a spectral basis and solve for the broad scale motion. This approach is well documented [e.g. *Bloxham and Jackson*, 1991] and our implementation follows that of *Celaya and Wahr* [1996] closely. We first decompose $\mathbf{u}$ into a sum of toroidal and poloidal vectors with corresponding scalars T and S:

$$\mathbf{u} = \mathbf{u}_T + \mathbf{u}_S = \nabla_s \times (\mathbf{r}T) + \nabla_s (rS). \tag{3}$$

These scalar functions together with B_r are then each expanded in terms of Schmidt quasi-normalized complex spherical harmonics $Y_l^m(\theta, \phi)$, weighted by complex coefficients. The series for B_r, T and S are truncated at L_B, L_u and L_u respectively. Thus,

$$\begin{aligned} B_r &= \sum_{l=1}^{L_B} \sum_{m=-l}^{l} (l+1) \left(\frac{a}{r}\right)^{l+2} f_l^m(t) Y_l^m(\theta, \phi), \\ T &= \sum_{l=1}^{L_u} \sum_{m=-l}^{l} \tau_l^m(t) Y_l^m(\theta, \phi), \\ S &= \sum_{l=1}^{L_u} \sum_{m=-l}^{l} s_l^m(t) Y_l^m(\theta, \phi), \end{aligned} \tag{4}$$

where the complex coefficients f_l^m are related to the usual Gauss coefficients by $f_l^m = \frac{1}{2}(g_l^m - ih_l^m)$ and $r = a$ at the Earth's surface. To convert (2) to its spectral form we simply substitute (3) and (4) for their equivalents in (1), multiply through by $Y_l^{m*}(\theta, \phi)$ ($*$ denotes complex conjugation) and integrate over the CMB. This leaves the matrix equation,

$$\dot{\mathbf{f}} = \mathbf{A}\mathbf{v}, \tag{5}$$

which forms the basis of the inverse problem to be solved. The matrix $\mathbf{A}$ is a function of the main field Gauss coefficients f_l^m weighted by Gaunt or Elsasser integrals, $\dot{\mathbf{f}}$ contains the secular variation (SV) of the core radial field, and $\mathbf{v}$ is a column vector of the unknown toroidal and poloidal flow coefficients we seek. Both $\mathbf{A}$ and $\dot{\mathbf{f}}$ are assumed to be known exactly. Details of the integration leading to (5) entail straightforward but tedious algebra [see *Whaler*, 1986; *Voorhies*, 1986; *Bloxham*, 1988].

We seek a steady solution $\mathbf{v}$ to (5) which minimizes, in the spectral domain, the difference between observed and predicted SV at Earth's surface over the time between t_1 and t_2 (as noted above we will choose $t_2 - t_1 = 10$ years). That is, we minimize,

$$\int_{t_1}^{t_2} [\dot{\mathbf{f}}^{\text{obs}} - \dot{\mathbf{f}}^{\text{pre}}]^{*T} \mathbf{W} [\dot{\mathbf{f}}^{\text{obs}} - \dot{\mathbf{f}}^{\text{pre}}]\, dt \tag{6}$$

where for equal weighting $w_{ij} = \frac{(2-\delta_{m,0})}{2l+1}\delta_{ij}$ (the w_{ij} differ from 1 only because the Y_l^m are not fully normalized), and the superscript T denotes matrix transpose. The predicted SV, $\dot{\mathbf{f}}^{\text{pre}}$, is a product of the unknown flow $\mathbf{v}$ and the main field matrix $\mathbf{A}$, itself a function of $\mathbf{v}$,

$$\dot{\mathbf{f}}^{\text{pre}} = \mathbf{A}(\mathbf{v})\mathbf{v}. \tag{7}$$

Substitution of (7) for $\dot{\mathbf{f}}^{\text{pre}}$ in (6) poses a minimization problem non-linear in the unknown $\mathbf{v}$. To linearize (6), $\mathbf{A}(\mathbf{v})$ is replaced by an equivalent matrix $\mathbf{A}$ formed with known time-varying main field Gauss coefficients determined at r=a and downward continued to the CMB. Thus, instead of (6) we minimize

$$\int_{t_1}^{t_2} [\dot{\mathbf{f}}^{\text{obs}} - \mathbf{A}\mathbf{v}]^{*T} \mathbf{W} [\dot{\mathbf{f}}^{\text{obs}} - \mathbf{A}\mathbf{v}]\, dt. \tag{8}$$

However, small errors in f_l^m and ∂f_l^m observed at the Earth's surface are greatly amplified when the field is downward continued to the core boundary, especially for large values of l and m. To prevent the magnified noise from distorting $\mathbf{v}$, the inversion is typically damped. A common means of damping [see, e.g., *Voorhies*, 1993; *Jackson et al.* 1993] is to seek solutions which minimize lateral gradients in the flow radial vorticity,

$$\Gamma = (\hat{\mathbf{r}} \times \nabla_s) \cdot \mathbf{u}, \tag{9}$$

as well as in its horizontal divergence,

$$D = \nabla_s \cdot \mathbf{u}. \tag{10}$$

This choice tends to bias the inversion in favor of the broad scale solution. Consequently, we seek flows which simultaneously minimize (8) as well as

$$\frac{c^4}{\pi} \int (\nabla_s D)^2 + (\nabla_s \Gamma)^2 d\Omega =$$

$$\sum_l \frac{l^3(l+1)^3}{2l+1} \sum_m [(\tau_l^m)^2 + (s_l^m)^2] = \mathbf{v}^{*T}\mathbf{N}\mathbf{v}, \tag{11}$$

where Ω is the whole CMB surface and $\mathbf{N}$ is the diagonal damping matrix. The corresponding solution vector of complex toroidal and poloidal flow coefficients is given by,

$$\mathbf{v} = \left[\int_{t_1}^{t_2} \mathbf{A}^{*T}\mathbf{W}\mathbf{A}\, dt + \lambda_d \mathbf{W}\right]^{-1} \left[\int_{t_1}^{t_2} \mathbf{A}^{*T}\mathbf{W}\dot{\mathbf{f}}\, dt\right]. \tag{12}$$

This is the unweighted, damped, linear, least squares, steady solution. The damping parameter λ_d is adjusted to find a suitable balance between a smooth solution and one that fits the data closely.

To solve the steady non-linear problem, *Bloxham* [1988] first obtains an initial estimate of $\mathbf{v}$ via the linear problem, and then adds corrections $\mathbf{v}'$ by updating $\mathbf{A}(\mathbf{v})$ with the new $\mathbf{v}$ and repeating the inversion until these corrections converge. In the balance of this analysis the simpler linearized inversion is preferred because over a ten year time-span the non-linear inversion appears to only negligibly change the outcome.

4. Radial Field Models

To construct $\mathbf{A}$ and $\dot{\mathbf{f}}$ we use the radial field models "ufm1" (1840 - 1990) and "ufm2" (1690 - 1840) of *Bloxham and Jackson* [1992]. With recent data ufm1 has been updated to 1997. Both models are expanded to degree and order 14 in space with spherical harmonics. Their time dependence is represented using a cubic B-spline basis.

The dataset used to construct these models contains no measurements of the absolute intensity of the field prior to 1840. Nevertheless, the declination and inclination data recorded between 1690 and 1840 can be used to determine the ratio of every Gauss coefficient f_l^m to the axial dipole term f_1^0 using a method originally proposed by *Bauer* [1894] and more recently described by *Barraclough* [1973]. The viability of this approach has also been evaluated by *Hulot et al.* [1997].

With no further information we can only determine the intensity of one part of the field relative to another at the same epoch. A comparison between field values from two different epochs is meaningless if f_1^0 changes significantly in the interim. But how fast does f_1^0 change, and can its variation be modeled in some simple way? A plot (Figure 3) of the axial dipole during epochs for which there is an adequate distribution of intensity data (from 1840 onwards) shows a nearly linear variation that sums to no more than about 8% over 150 years ($\approx$ 0.05% per year). Provided the rate of change of f_1^0 has been similar to this for the last 310 years, a linear extrapolation of the axial dipole data provides estimates of the missing f_1^0 intensity for the 18$^{\text{th}}$ and 19$^{\text{th}}$ centuries. This has been the approach followed by several authors [*Braginskii and Kulanin*, 1971; *Braginskii*, 1972; *Barraclough*, 1973]. In developing ufm2, *Bloxham and Jackson* [1992] take a similar tack.

5. Flow Estimates and CAM

To construct smoothly varying time-dependent models of core flow we invert estimates of $B_r(a, \theta, \phi, t)$ and $\partial_t B_r(a, \theta, \phi, t)$ for steady motions within a 10-year sliding window moving from 1690 to 1997. Each flow solution is assigned to the central time of the window, and the window is advanced 2.5 years before recomputing the flow. Consecutive solutions are not fully independent (only every 5th solution is), but we have chosen a 2.5 year time-step so that the corresponding CAM series has a sampling interval reasonably close to that of the LOD series. Shortening the width of the 10-year sliding window is not a viable alternative as widths much less than 10 years threaten to make $\mathbf{A}^{*T}\mathbf{WA}$ a singular matrix. *Jackson et al.* [1993] also construct a time-dependent motion (for the period 1840-1990) by individually computing flows at 2.5 year intervals, although their solutions are constrained to be geostrophic rather than steady. A more sophisticated means of resolving time-varying flow has been presented by *Jackson* [1997a] who constructs his flows on a temporal basis of B-splines, erected over the 150 years spanned by ufm1. However, his resulting estimates of CAM differ little from that of *Jackson et al.* [1993] suggesting the approach we adopt here is probably sufficient for a decade-scale comparison with the LOD data.

Initially we inverted for steady flows with the damping parameter λ_d set to 10^{-3}. This yielded solutions capable of explaining 97.4% of the observed variance in $B_r(a, \theta, \phi, t)$ between 1840 and 1997, and 96.6% of the variance between 1690 and 1840. Inspite of their success in accounting for nearly all the main field variance, when these same motions were used to infer flow throughout the core following the approach of *Jault and Le Mouël* [1989], the resulting CAM since 1840 made a rather poor match to the LOD, even over the last 60 years (Figure 4). Damping the inversion more heavily ($\lambda_d = 10^{-2}$) did not appreciably change this result.

The much better match between relative changes in CAM and LOD appearing in Figure 1 was achieved with the flow models of *Jackson et al.* [1993] who assume surface motions are strictly geostrophic to resolve the non-uniqueness in (2). Figures 1 and 4 raise the issue of whether a good correlation between CAM and LOD necessarily implies that the flow near the CMB is tangentially geostrophic. Our results would seem to indicate that it does. However, *Holme and Whaler* [1998] have considered this issue and report that indeed other constraints on the inversion (such as that the flow is purely toroidal) can lead to a CAM-LOD correlation nearly as good as what is obtained with geostrophy.

Imposition of the geostrophic constraint is not needed to obtain formally unique core surface motions; the steady motions constraint already ensures that. Nevertheless, weighting the solution toward a more nearly geostrophic flow does significantly improve the CAM-LOD correlation, especially since about 1930, while still accounting for over 90% of the main field variance. A strong correlation between CAM and LOD might not always be observed, but because the most recent data are also the most reliable and the strong post-1930 correlation disappears without geostrophy, we proceed under the supposition that core motions must be geostrophic to some degree.

Thus, we revise our approach to seek steady solutions which simultaneously minimize the mean square ageostrophy of the flow,

$$\frac{c^2}{4\pi}\int [\nabla_s \cdot (\mathbf{u}\cos\theta)]^2 d\Omega = (\mathbf{G}\mathbf{v})^{*T}\mathbf{W}\mathbf{G}\mathbf{v}, \quad (13)$$

the prediction error (8), and the mean squared gradients of flow vorticity and horizontal divergence (11). The solution vector now takes the form,

$$\mathbf{v} = \left[\int_{t_1}^{t_2} \mathbf{A}^{*T}\mathbf{W}\mathbf{A}\, dt + \lambda_d \mathbf{W} + \lambda_g \mathbf{G}^{*T}\mathbf{W}\mathbf{G}\right]^{-1} \times \left[\int_{t_1}^{t_2} \mathbf{A}^{*T}\mathbf{W}\dot{\mathbf{f}}\, dt\right], \quad (14)$$

where the scalar factor λ_g determines the degree of compliance with the geostrophic constraint.

For the period between 1840 and 1992 several trial inversions were conducted using different values of λ_d and λ_g. For $\lambda_d = 10^{-3}$ and $\lambda_g = 10^8$ we obtain smoothly varying solutions that explain 93.5% of the variance in $B_r(a, \theta, \phi, t)$ for the period 1840 to 1992, and 91.7% of the variance between 1690 and 1840. These solutions generate a combined (1690-1992) CAM series that matches the LOD well after about 1930, but that good match gradually deteriorates going backwards in time before that date (Figure 5). The CAM results after 1840 are nearly identical to those of *Jackson et al.* [1993] and *Jackson* [1997a].

6. LOD vs. CAM Before 1840

The new result reported here is an extension of the CAM series beyond 1840 to 1690 as it appears in Figure 5. The LOD series is remarkably flat across the 170 years preceding 1850 in stark contrast to its behavior during the subsequent 145 years. Yet the CAM over this time does not track the LOD series. Rather it exhibits non-periodic changes somewhat larger than those seen after 1860. The large peak at about 1755 is nearly twice as large as the peak at 1900. To be sure, the excellent correlation between CAM and LOD already begins to wane going back in time before about 1930. But in moving from 1930 back to 1850 it is clear that both series continue their oscillatory behavior. In fact, the disparities during this time seem to reflect more a difference in phase than anything else. Except for the dip in CAM at about 1835, the same might be said about the CAM-LOD correlation going back to 1800 or even 1780. Before then, however, the series diverge.

The uncertainties of the 18$^{\text{th}}$ and 19$^{\text{th}}$ century LOD data estimated by *Stephenson and Morrison* [1984] do not exceed ± 1 ms. These formal errors (equivalent to 1 standard deviation) represent only about 8% of the 13 ms peak to trough amplitude of the CAM peak centered on 1755. By this measure the disparity between LOD and CAM during the 18$^{\text{th}}$ and early 19$^{\text{th}}$ centuries appears significant. Re-reductions of the lunar occultation measurements using improved lunar ephemerides and star catalogs yield improved LOD determinations, but such corrections are typically smaller than 1 ms and usually affect LOD periods much shorter than a decade [see, for example, *Morrison*, 1979]. In fact, neglecting for the moment errors in the CAM series, the uncertainty in the LOD could stand to be several times larger without diminishing the significance of the pre-1840 LOD-CAM discrepancy.

However, the uncertainty in the CAM series before 1840 is much harder to quantify. In strict terms, errors for B_r and $\partial_t B_r$ cannot be estimated before 1840 because of the lack of intensity data. For this reason there are no uncertainties available for model ufm2. At best only errors for the ratios f_l^m / f_1^0 ($l = 1, 2, \ldots$) can be estimated. Nevertheless, to secure a lower bound on the actual standard errors for the $f_{l>1}^m$ we could take advantage of the method used to estimate field intensity before 1840 (see section 4). Recall that the procedure involved extrapolating a linear fit made to $f_1^0(t)$ after 1840 to estimate the values of $f_1^0(t)$ before that time. If we assume that $f_1^0(t)$ during the 18$^{\text{th}}$ and 19$^{\text{th}}$ centuries behaves *exactly* as predicted, the standard errors for the $f_{l>1}^m$ are easily computed. This was the approach adopted by *Barraclough* [1974] who produced low degree ($l \leq 4$) field models at 50 year intervals between 1600 and 1850 that included standard deviations for every Gauss coefficient (save, of course, for f_1^0). We consider these a lower bound as they do not account for any error in estimating f_1^0 itself.

Unfortunately the errors reported by *Barraclough* are quite large. At 1700, 1750, 1800, and 1850 the errors averaged over the 24 Gauss coefficients at each epoch exceed 42%, 19%, 15% , and 13%, respectively. Although uncertainties of this magnitude are perhaps not too disturbing for main field coefficients, they dwarf all estimates of the low degree SV as modeled by ufm2. Resolving the core surface flow is particularly sensitive to the SV. If *Barraclough's* errors are even remotely indicative of how imprecisely we know the main field (and by first differences its rate of change) before 1850, then the flow and CAM during that time are indeed very poorly constrained - even when we assume f_1^0 is known exactly.

7. Estimating Uncertainty in CAM

At a minimum what would we need to place useful constraints on CAM prior to 1840? There is little hope of ever overcoming the lack of early magnetic intensity data. Nevertheless, the linear extrapolation of f_1^0 back to 1690 does not appear entirely unreasonable given the nearly linear behavior of f_1^0 after 1840 (Figure 3). The post-1840 axial dipole data are not known well enough to warrant a higher order fit, but if they were, would it make any difference? Quadratic and cubic fits to the f_1^0 data extrapolated back to 1690 do lead to some differences (Figure 3) in the axial dipole intensity when compared to the linear fit. As expected, however, the *relative* changes in f_1^0 are still much smaller and of lower frequency than relative changes in the higher degree field; the concomitant effect on relative CAM changes will be negligible. Of course this does not rule out the unlikely possibility that f_1^0 did fluctuate wildly during the 18^{th} century, but there is no evidence or reason to think this occurred.

The far greater obstacle is our poor knowledge of the higher degree field's rate of change. Clearly more accurate estimates of the secular variation are necessary. This might be an impossible request if we demand accurate knowledge of the SV coefficients $\partial_t f_l^m$ to high degree and order. The poor spatial distribution and sparseness of magnetic data recorded before 1840 make it difficult to constrain the small scale field with sufficient accuracy.

Yet, *Jault et al.* [1988] have shown that under the special restrictions they impose to resolve deep core flow, the angular momentum of the whole core is carried in the values of just two flow modes, τ_1^0 and τ_3^0, so that,

$$J_z = \frac{4\pi c^4 \rho}{15}\left(\tau_1^0 + \frac{12}{7}\tau_3^0\right), \tag{15}$$

where c is the core radius. All other modes are orthogonal to the kernel of integration in (1).

In general every flow coefficient (toroidal or poloidal) depends on all Gauss coefficients (f_l^m and $\partial_t f_l^m$) as is evident from (12) or (14). This would seem to preclude the possibility that we could rely exclusively on the low degree ($l \leq 4$) field to determine the low degree flow modes responsible for the angular momentum. However, when we truncate the ufm2 and ufm1 fields to $L_B = 4$ and invert for steady geostrophic flows truncated to $L_u = 3$ we obtain solutions that in turn produce a CAM series (Figure 6) similar to that obtained using the full ($L_B = 14$) fields and high degree ($L_u = 10$) flows. This result suggests that in fact we can depend on the broad scale field to reliably approximate long term changes in CAM. This means our request for accurate Gauss coefficients and their SV can be limited to exactly those low degree coefficients that historical recordings are most likely to resolve well.

How precisely, then, must these degree 4 and less Gauss coefficients be determined to adequately constrain the $L_u = 3$ flow and its associated angular momentum? To answer this we have truncated model ufm2 to degree and order 4 and perturbed the remaining complex Gauss coefficients f_l^m in (4) with varying levels of noise. To each coefficient we add a random complex number ϵ (where $-1-i \leq \epsilon \leq 1+i$), scaled by a fixed percentage x of the coefficient magnitude. At every time step a new random number is used. Thus,

$$[f_l^m(t)]_{\text{noisy}} = f_l^m(t)\left[1 + \frac{x}{100}\epsilon(t)\right]. \tag{16}$$

First differences of the $[f_l^m(t)]_{\text{noisy}}$ provide the corresponding noisy SV. The percent noise $x/100$ is changed with every new trial to explore the propagation of main field errors of different magnitudes.

Figure 7 illustrates the results of several trials where the percent noise added to the main field increases from 0% to 10%. For each trial we compute the deviations from the unperturbed CAM at every epoch and then average these to find the mean distortion in CAM (measured in ms) incurred by the x percent noise added to B_r. When the main field noise approaches 10% the CAM is only known to within about 6.6 ms, on average. At this level of noise the large peak in CAM at around 1765 is no longer significant. These noise trials also confirm our earlier statement that the main field uncertainties reported by *Barraclough* [1974] (all of which exceed 13%) render pre-1840 CAM practically an unknown quantity, the missing intensity data notwithstanding.

Is there any hope of ever determining the first 24 pre-1840 Gauss coefficients to within an average of 10% or less? Currently Andrew Jackson and colleagues are in the process of culling additional magnetic measurements from historical maritime traffic records to improve models of the field between the 17^{th} and 19^{th} centuries [*Jackson et al.*, 1997b]. To place the 10% average main field error bar in context we could compare this level of noise with the errors reported for the degree 14 model ufm1 which spans the time after 1840. Such a comparison is not entirely straightforward because the magnitude of the error on individual coefficients depends in part on the number of coefficients resolved in the field model inversion. Coefficients of a low degree field model will tend to have larger uncertainties because the misfit is divided up among fewer model parameters. The errors for field model ufm1 at 1850 sum to 237.6 nT. If we divide this total misfit among the 24 coefficients of a $L_B = 4$ model we find an average error of about 9.9 nT per Gauss coefficient. This represents an average uncertainty of about 3.5%, which, by Figure 7, corresponds to an error in CAM of between 2.0 and 3.3 ms. Thus, in principle at least, it might be possible to achieve the desired level of accuracy.

8. Discussion

A central issue in this investigation is the fidelity of the LOD record and the magnetic field measurements from the 18^{th} and early 19^{th} centuries. If we trust neither data set before 1840 then nothing more can be added to what has already been said about the data recorded since that time. If only the pre-1840 magnetic data are believed, then it appears we may conclude that the large scale core flow (and its angular momentum) has continuously exhibited decadal accelerations since at least 1690. If in fact this variability results from torsional oscillations between concentric cylindrical shells of fluid then these may be a long term feature of core flow. These conclusions are perhaps not too surprising.

If, however, we trust only the LOD record before the mid-19^{th} century then the matter becomes more intriguing. We would like to know why there suddenly appeared decade swings in the LOD record around 1850 after nearly 150 years of relative dormancy. This was the impetus behind our report. As noted in the introduction, excluding the core leaves little else to provide a satisfactory explanation for decadal LOD changes, so we have no choice but to entertain its role even though we may not believe the magnetic data. If indeed core motions are responsible and the LOD record is accurate then somehow a fundamental change in the core and/or its coupling to the mantle must have occurred during the mid-19^{th} century. Perhaps core-mantle coupling really has remained consistent over time yet for some reason core motions were not variable on the decade time scale before the mid-19^{th} century. In this case we should expect a reliable pre-1850 CAM series to be equally as dormant as the LOD was. A lack of decadal variability in CAM prior to 1850 would then indicate that for some unknown reason torsional oscillations of the broad scale core flow vanished between 1690 and 1850. This indeed would be surprising.

It is true that the ambiguity in the early magnetic field limits the potential of the 1850 LOD change to reveal more about the Earth's deep interior. Nevertheless, the CAM series appearing in Figure 5 represents an estimate based on the magnetic field model that *best fits* the data compiled by *Bloxham and Jackson* [1992]. If the additional data collected by *Jackson et al.* [1997b] succeed in reducing pre-1840 field uncertainty, but do so without appreciably changing the ufm2 main field coefficients, the discrepancy between LOD and CAM in the 18^{th} and 19^{th} centuries will only be reinforced. This discrepancy reaches up to 5 and sometimes 7 ms between 1700 and 1750 which would correspond to a nearly 10% average error in the low degree radial main field, if in fact coupling was effective at the time. Revised estimates of the main field would have to differ from ufm2 by about this much to accommodate a match to the observed early 18^{th} century LOD.

If the new magnetic data fail to close this rather large gap we may be forced to consider the case where *both* LOD and magnetic records are trustworthy. A possible implication is that LOD has not always been a good indicator of CAM because core-mantle coupling was ineffective before the mid-19^{th} century. It may be that decadal variability only became a characteristic of LOD once the mantle coupled to the core starting about 150 years ago. In that case we expect to see (as we do in Figure 5) the CAM and LOD series diverge before 1850 (barring a completely fortuitous match unrelated to coupling). What might this say about the nature of core-mantle coupling? If fluid motions have truly been continuously variable since 1690 as the magnetic data indicate, then it seems doubtful that topographic or gravitational torques could explain an intermittent coupling. After all, it is hard to imagine how CMB topography or the lower mantle density distribution could have changed significantly around 1850 to suddenly lock the mantle to the core after 150 years of independent motion. Rather, such evidence might tend to favor coupling by a variable electromagnetic torque that at times could have been too weak to transfer momentum despite an accelerating flow. Before 1850 the net electromagnetic coupling may have been small as a consequence of near cancellation between torque produced by a changing field in the conducting lower mantle and an opposite torque due to electric currents leaking outwards from the core [see *Stix and Roberts*, 1984].

Distinguishing between these scenarios could potentially be of great use in the ongoing effort to understand core flow, core-mantle coupling, and historical Earth rotation. For the moment we await the results of *Jackson et al.* hoping they will have their intended effect.

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Figure 1. Relative changes in the observed LOD versus LOD changes predicted by variations in CAM. A 1 ms difference in the predicted LOD requires a $6 \times 10^{25}\mathrm{kgm^2s^{-1}}$ change in core momentum. Observed LOD values shown between 1840 and 1962 are those reported by *McCarthy and Babcock* [1986]; after 1962 we show the values of *Gross* [1999] smoothed to dampen large seasonal variations. A trend of 1.7 ± 0.1 ms/cy was removed from both observed LOD series to account for tidal braking of the Earth's spin (2.3 ± 0.1 ms/cy) and changes in the Earth's polar moment of inertia associated with post-glacial rebound (-0.6 ± 0.1 ms/cy). Core flow models of *Jackson et al.*, [1993] were used to generate the predicted LOD. Fluid motion within a cylinder tangent to the inner core was neglected in the CAM calculation.

Figure 2. Relative changes in the LOD reported by *Stephenson and Morrison* [1984] and *McCarthy and Babcock* [1986] and *Gross* [1999]. These series were detrended as described in caption to Figure 1. *Stephenson and Morrison* report a 1 ms uncertainty in LOD between 1700 and 1800 which thereafter decreases steadily with time.

Figure 3. Extrapolated linear, quadratic, and cubic fits to changes in the axial dipole since 1840.

Figure 4. LOD versus CAM when the core surface flow is constrained to be piecewise steady (but not geostrophic). The LOD data are those shown in Figure 2.

Figure 5. LOD versus CAM between 1690 and 1992 when the core surface flow is constrained to be piecewise steady and geostrophic.

Figure 6. CAM obtained with field model ufm1 truncated to degree 4 and flow models truncated to degree 3 (dash-dot), versus CAM derived with the full degree 14 ufm1 field and flows truncated to degree 10 (solid).

Figure 7. Average uncertainties in pre-1840 CAM resulting from the propagation of errors in the main field that vary between 0 (solid line) and 10% of the magnitude of the Gauss coefficients they perturb.

Fig. 1

Historical Length of Day Changes

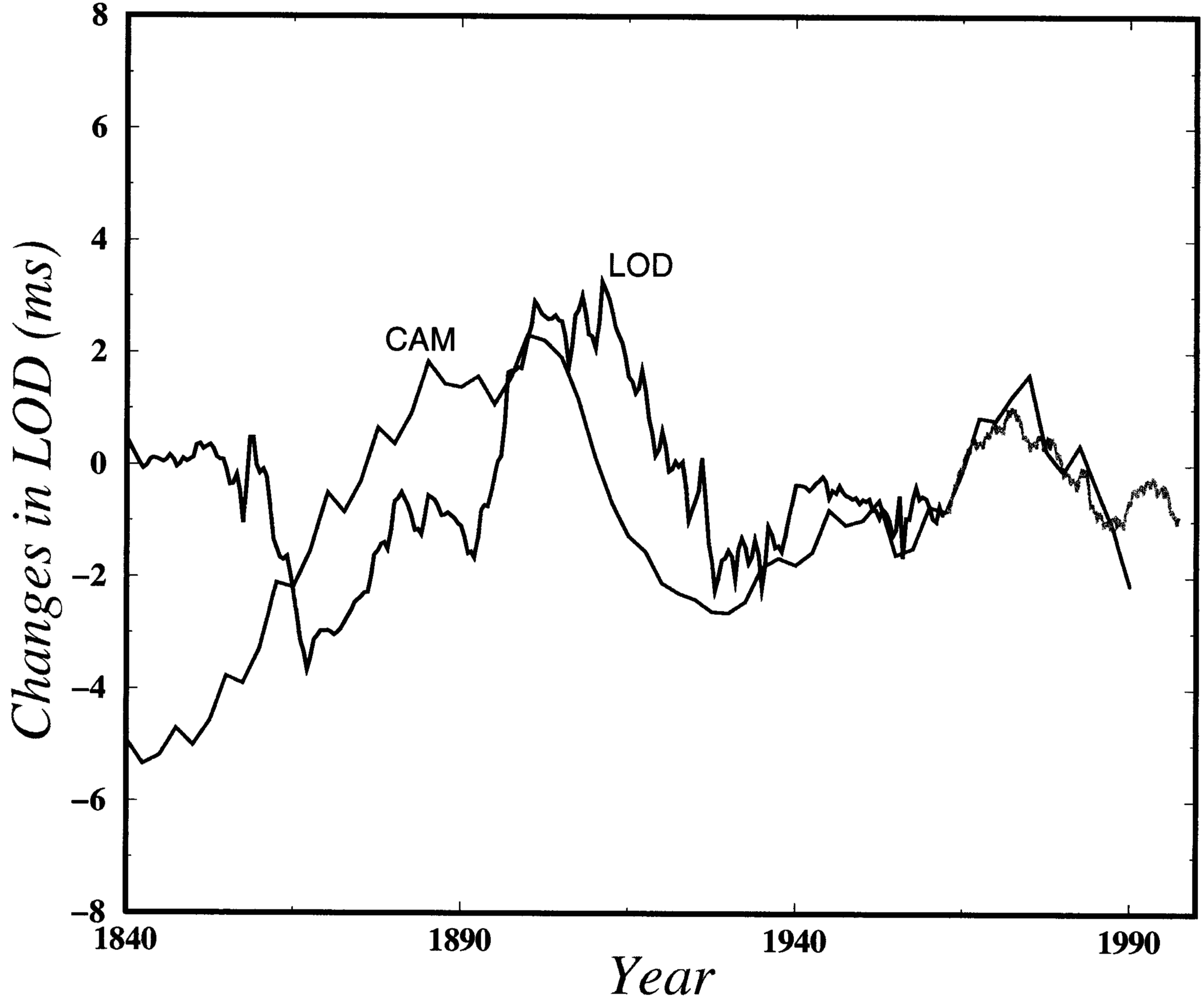
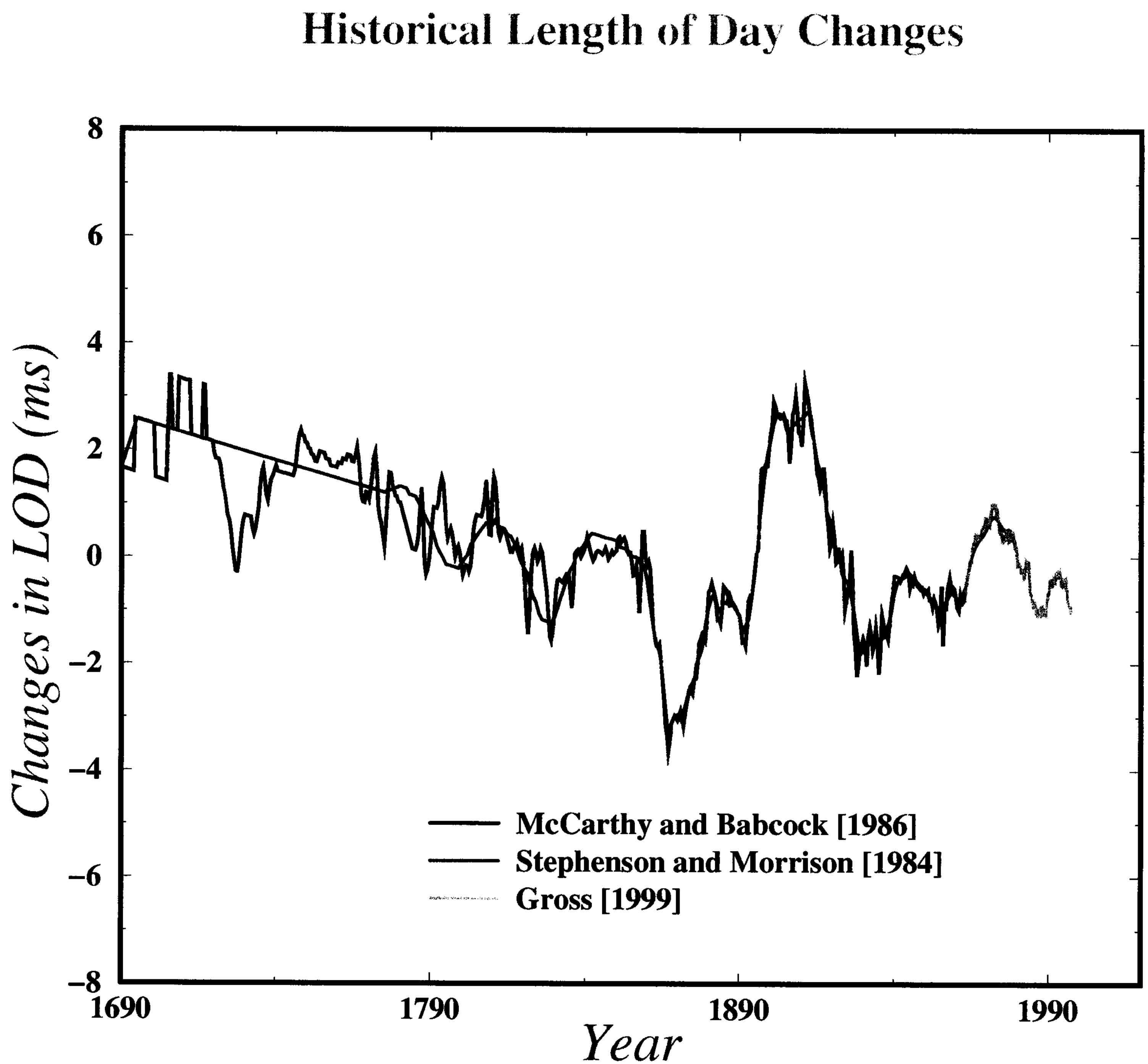
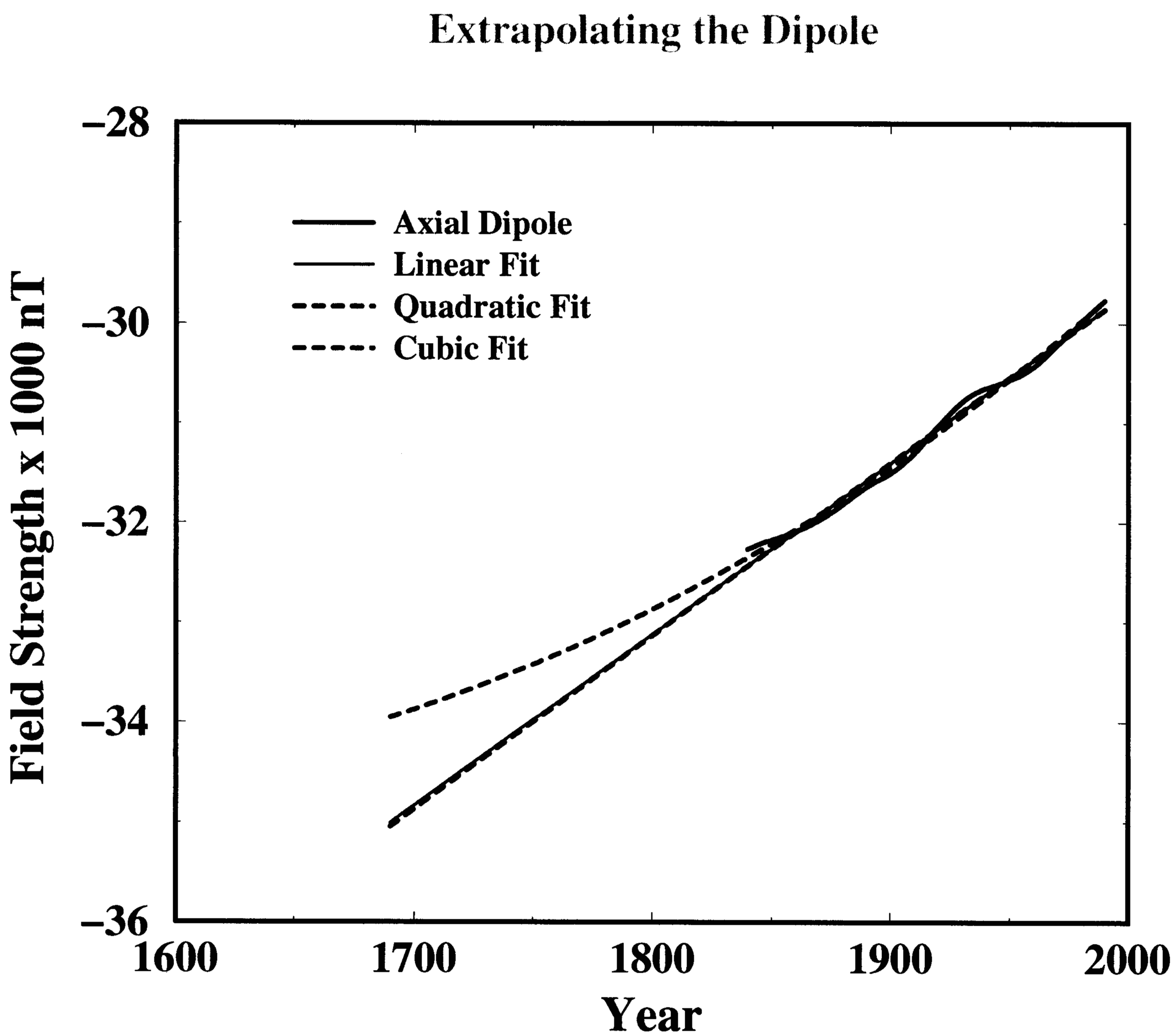


Fig. 2



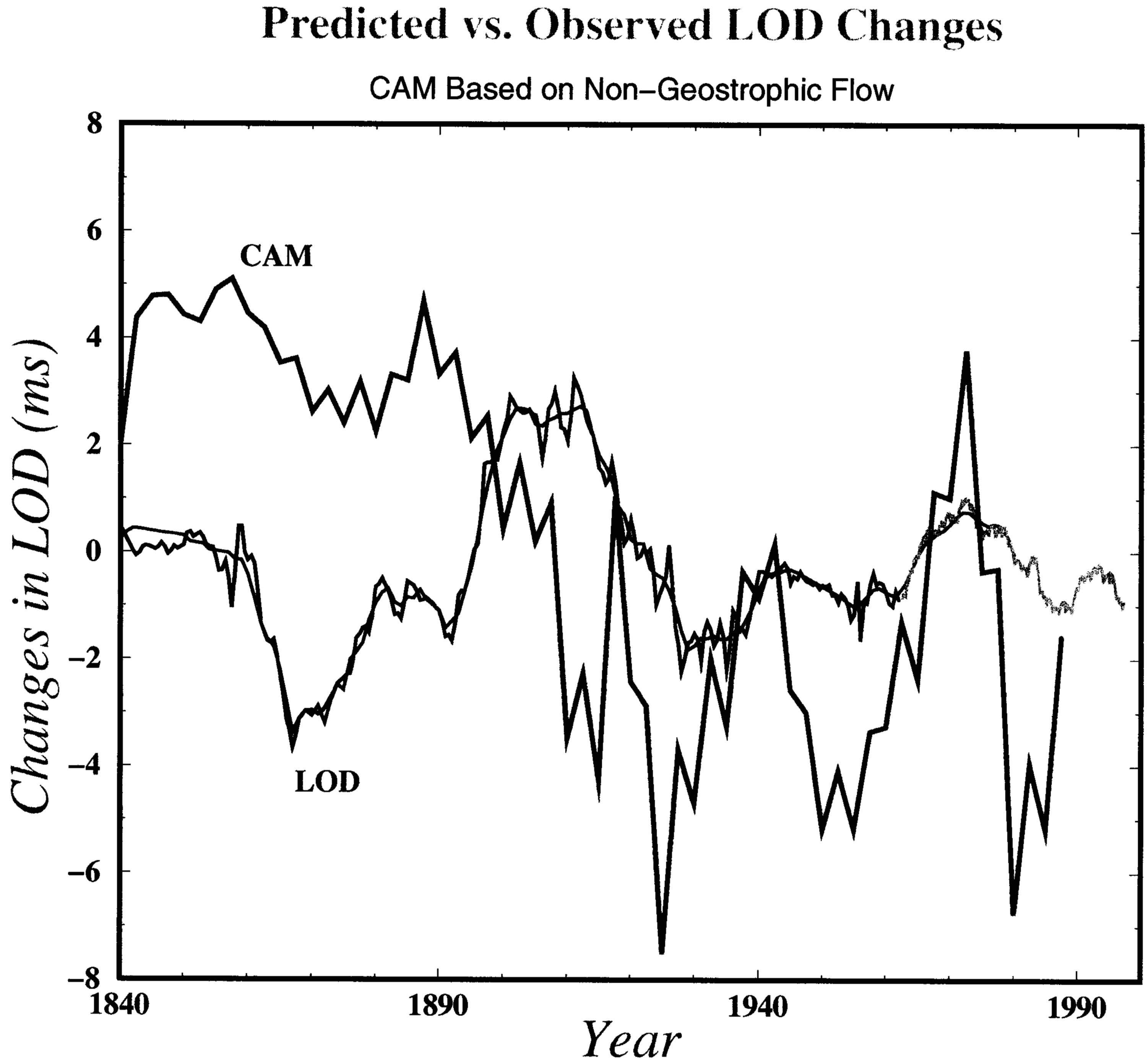
Historical Length of Day Changes
Changes in LOD (ms)
8
6
4
2
0
−2
−4
−6
−8
1690
1790
1890
1990
Year
McCarthy and Babcock [1986]
Stephenson and Morrison [1984]
Gross [1999]

Fig. 3



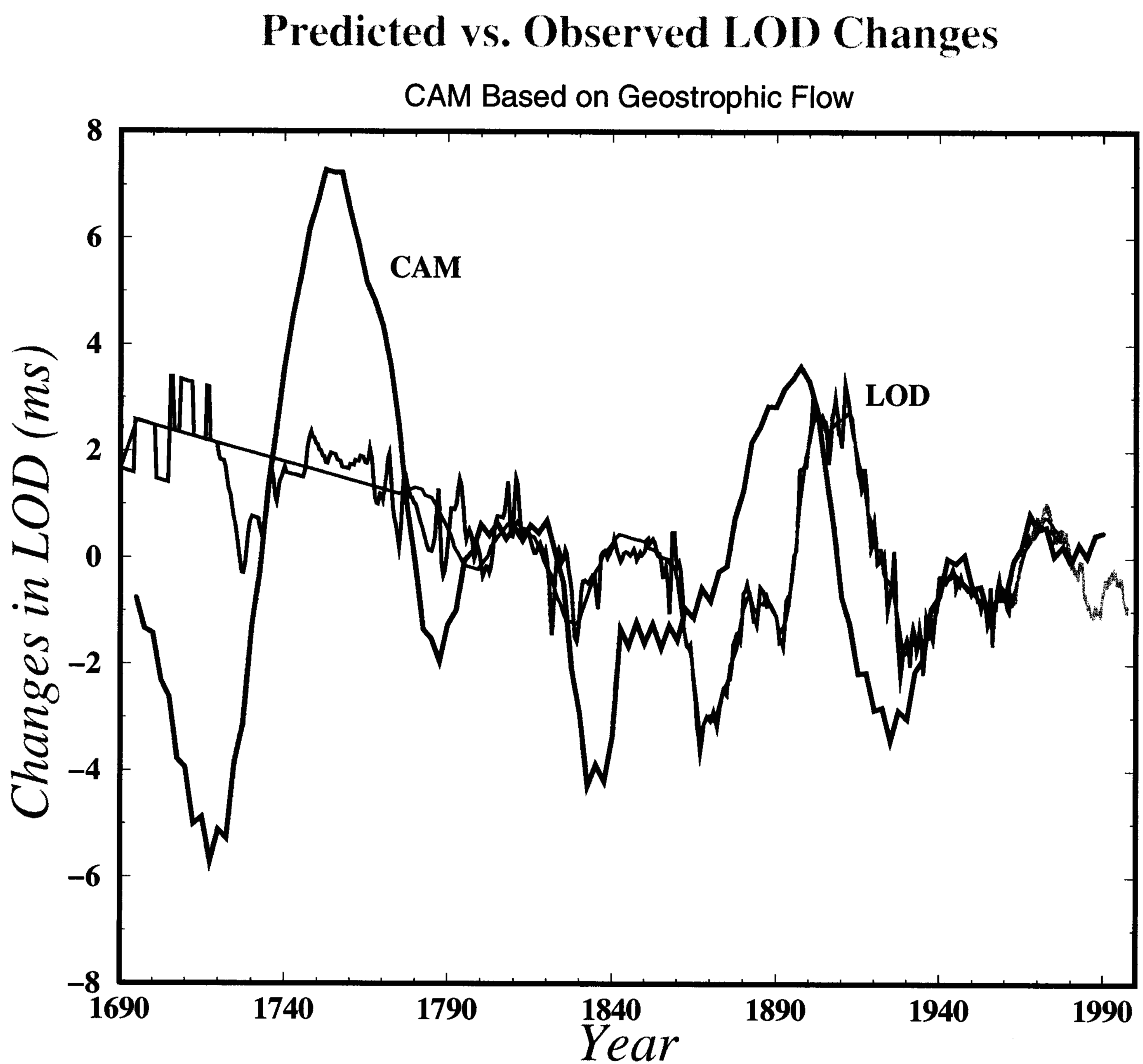
Extrapolating the Dipole
Axial Dipole
Linear Fit
Quadratic Fit
Cubic Fit
-28
-30
-32
-34
-36
Field Strength x 1000 nT
1600
1700
1800
1900
2000
Year

Fig. 4



Predicted vs. Observed LOD Changes
CAM Based on Non–Geostrophic Flow
CAM
LOD
Changes in LOD (ms)
8
6
4
2
0
−2
−4
−6
−8
1840
1890
1940
1990
Year

Fig. 5



Predicted vs. Observed LOD Changes
CAM Based on Geostrophic Flow
CAM
LOD
Changes in LOD (ms)
Year
8
6
4
2
0
−2
−4
−6
−8
1690
1740
1790
1840
1890
1940
1990

Fig. 6

Comparing Different CAM Estimates

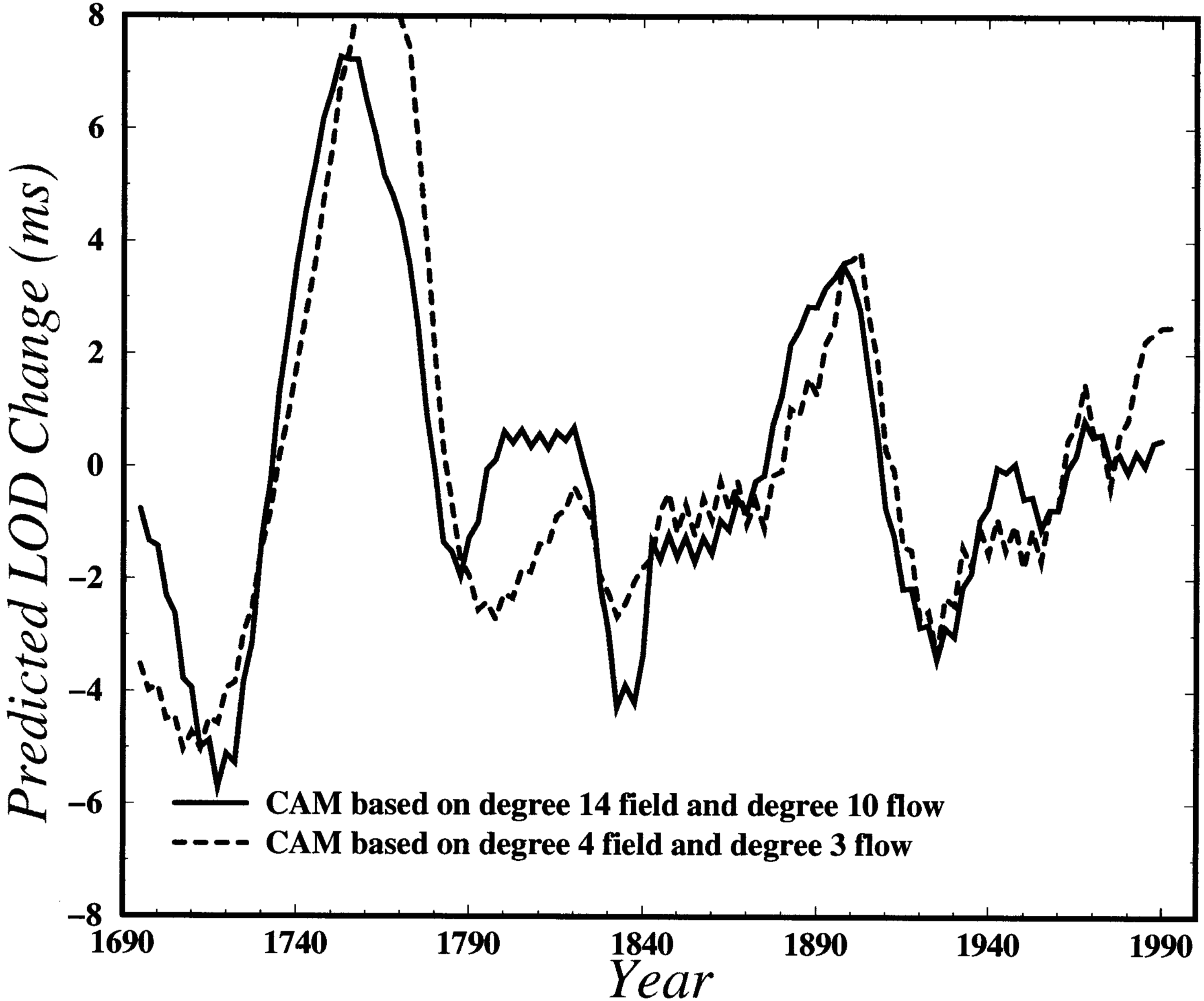


Fig. 7

Effect of Magnetic Field Errors on CAM